Static and dynamic behavior of autocatalytic replicators in heterogeneous reactor networks.

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1 x 1

Bifurcation analysis

I reactors

1 x 2

1 x N

Current work

1 x 3

2 x 2

Dynamic exploration

2 x 3

N x 1

N species

(Gray-Scott)

(Birol et al)
Reactor Network

\[ R + 2P_n \xrightarrow{k_n} 3P_n \]
\[ P_n \xrightarrow{d_n} D \]

\[ \frac{dr_i}{dt} = - \sum_{n=1}^{N} k_n r_i p_{in}^2 + f(1 - r_i) + g(r_{i-1} + r_{i+1} - 2r_i) \]

\[ \frac{dp_{in}}{dt} = k_n r_i p_{in}^2 - p_{in}(f + d_n) + g(p_{i-1,n} + p_{i+1,n} - 2p_{in}) \]

- resource concentration
- resource concentration in feed
- species \( n \) concentration
- dead species
- \( R/R_0 \) in reactor \( i \)
- \( P/R_0 \) in reactor \( l \)
- growth rate of species \( n \)
- death rate of species \( n \)
- feed flow rate
- interaction flow rate
- time
Reactor Network

Diagram showing a network of reactors with flow directions indicated by arrows. The diagram includes labels such as $R_0$, $R$, $P$, $D$, $f$, and $g$. The network consists of interconnected reactors with flow paths marked by dashed and solid lines.
Reactor Networks

Effects of increased spatial heterogeneity (large $I$)

- Complex behavior
- Steady states increase exponentially
- Coexistence of competitors
- Sustained oscillations
  - Phase locking: strong coupling
  - Quasiperiodicity: weak coupling
  - Traveling waves
- Emergence
  - Enlarge species existence space
Bifurcation Diagrams of resource concentration vs. feed flow rate for bi-directional ring networks of size 2-6 CSTRs

$k = 25$
$d = 0.1$
$g = 0.002$
Bifurcation Diagrams of species concentration vs. feed flow rate for bi-directional ring networks of size 2-6 CSTRs

- \( k = 25 \)
- \( d = 0.1 \)
- \( g = 0.002 \)
Steady state spatial patterns

Large networks contain steady state subsets of smaller network steady states

- 4-CSTR contains 2-CSTR SS
- 6-CSTR contains 3 and 2-CSTR SS
- 3,5,7-CSTR have unique SS → Prime numbers!
Bifurcation diagram for 3-CSTR bi-directional ring network

stable
unstable
Bifurcation diagram for 4-CSTR bi-directional ring network

H_3 → H_2 → H_1

Red: stable; blue: unstable
Bifurcation diagram for 4-CSTR uni-directional ring network

- Red: stable
- Blue: unstable
Bifurcation Diagrams of resource concentration vs. interconnection flow rate / feed flow rate (g / f) for bi-directional ring networks of size 2-7 CSTRs
Bifurcation Diagrams of resource concentration vs. interconnection flow rate / feed flow rate ($g/f$) for uni-directional ring networks of size 2-7 CSTRs
Reactor Networks

Flow rates must be controlled such that:

- Species are not purged from the network \((p_n=0)\)
- High flow rates remove species from the system before they can reproduce
- Low flow rates do not supply enough resource resulting in starvation
Agent Control Scheme

decision

observer

arbitration

reactor $i$

reactor $i + 1$
Agent Control Scheme

- Each agent has some level of control over the interaction of its neighbors
- Arbitrator receives operational requests from other agents
- Agents set max / min compromise values
- Agents may cooperate or compete
- Extinction is to be avoided at all costs by avoiding unstable regions
Agent Control Scheme

- A reactor may operate at one of many multiple states
- Agents try to improve operating conditions
  - Increase species concentration
  - Request assistance from neighbors
- Disadvantaged agent may negotiate for a more favorable outcome
Possible Event Outcomes

- High 1: Win
- Low 2: Lose

- High 1: Lose
- Low 2: Win

- High 1: Lose
- Low 2: Lose
Software Tools

- Gensym G2
- CVODE libraries

- KBS development shell
- Graphical interface
- Object oriented language
- Real-time intelligent applications

TCP/IP Bridge

CVODE ODE Solver

G2 Multi-Agent System
Sample Cases

Immediate retribution results in oscillation whenever one agent loses.

Allowing one loss per turn results in both agents increasing goals incrementally.
Conclusions

Reactor networks with autocatalytic replicators display a wide range of complex behavior including multi-stability, periodicity, quasi-periodicity and chaos.

A multi-layer agent-based system was developed for monitoring and control of spatially distributed processes and demonstrated via a case study of two competing neighbors in a network.

Competing agents can increase objective by accepting a one-turn loss.
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